

# A GENERALIZED TWO-DIMENSIONAL COUPLED-MODE ANALYSIS OF CURVED AND CHIRPED PERIODIC STRUCTURES IN OPEN DIELECTRIC WAVEGUIDES\*

Zong-Qi Lin\*\*, Shu-tong Zhou\*\*, William S. C. Chang, S. Forouhar and J. Delavaux

Department of Electrical Engineering and Computer Sciences  
University of California, San Diego  
La Jolla, California 92093

## ABSTRACT

A generalized two-dimensional coupled-mode analysis of curved and chirped quasi-periodic structures in planar dielectric waveguides has been formulated. This analysis can be used to design curved and chirped quasi-periodic structures for obtaining phase matched interaction between two specific guided-wave beams. Alternatively, it can be used to calculate the amplitude and the phase of the diffracted guided-wave beam for a given quasi-periodic structure and for a specific incident beam, including the effect of the phase mismatch.

Curved and chirped gratings in optical dielectric waveguides have been investigated by a number of researchers to obtain reflection, focusing, collimation, coupling, and Fourier analysis of guided-waves<sup>1-4</sup>. They will also be useful to millimeter wave applications since similar dielectric waveguides are used in that wavelength region. A typical example, a chirped curved grating lens, is illustrated in Figure 1. In this case, the planar guided-wave beam is diffracted into a focused beam with a diffraction efficiency of 90%.

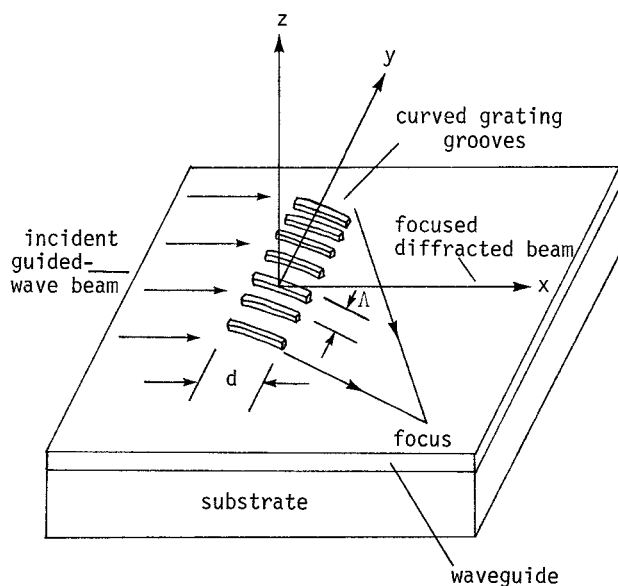


FIGURE 1: ILLUSTRATION OF A CURVED CHIRPED GRATING WAVEGUIDE LENS.

Chirped and curved gratings in optical waveguides have been analyzed by a number of researchers<sup>5,6</sup>. However, most of these analyses are based on perturbation methods or a modified plane-wave coupled-mode analysis<sup>5,6</sup>. The perturbation methods neglect the

feedback effect on the incident beam, hence they cannot be used to analyze volume interactions. The modified plane-wave coupled-mode analysis can only be used to analyze reflectors or structures that have a very limited chirping rate or curvature with no phase mismatch. Solymar has presented a general two-dimensional theory for phase matched diffraction in volume holograms<sup>7</sup>. We have extended his theory to vector wave equations and to analyze the volume interactions of two generalized guided-wave beams that may even include a limited amount of phase mismatch. In the example shown in Figure 1, these two beams consist of a plane guided-wave beam and a focused guided-wave beam.

In our analysis, we assume that there are only two guided-wave beams coupled to each other via the chirped or curved grating structure. These two beams can have a variety of horizontal variations such as plane, cylindrical or gaussian beams, but their  $z$  variation is always given by the mode profile of the planar waveguide. The grating will have a localized orientation and periodicity, i.e.  $\Lambda$ . The  $\Lambda$  varies slowly from one localized region of the grating to another. Within a localized region, the Bragg condition of diffraction is partially fulfilled by the  $\Lambda$  and two plane guided-wave representing the portion of the two beams in that localized region. The Q factor (given by  $2\pi\lambda_e d/\Lambda^2$  for plane guided-waves where  $\lambda_e$  is the effective wavelength of the guided-wave mode and  $d$  is the length of volume interaction) is high enough so that diffractions by the grating grooves into other guided-wave beams can be neglected. Our approach is to substitute an assumed form of the two guided-wave beams (with unspecified amplitude and phase variations) into the vector wave equation. When a generalized phase matching condition is satisfied, the vector wave equation is reduced to a coupled two-dimensional differential equation relating the amplitude and phase variations of the two beams. The generalized phase matching condition is used to design chirped and curved gratings that will give efficient diffraction for a given incident beam and a given desired diffracted beam. The solutions of the coupled differential equations plus boundary conditions are obtained numerically on a digital computer to give the amplitude and phase variations of the two beams as they emerge from the grating region. The numerical procedure is applicable both when the generalized

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\*\*Visiting Scholars from the People's Republic of China. Mr. Lin is from Chiao-Tung University and Mr. Zhou is from Fudan University, both in Shanghai.

phase matching condition is exactly satisfied and when there is a significant amount of phase matching. Such numerical solutions have been obtained for a large number of chirped and curved grating structures for various incident beam shapes and directions of propagation. From the numerical results, we have obtained information such as the diffracted field patterns, the diffraction efficiency, the angular range of the incident beam within which effective diffraction can take place, the effect of changing the beam shape of the incident beam, and the effect of changing the grating parameters on chirped and curved grating lenses in open planar dielectric waveguides.

The derivation of our analysis, examples of grating design based upon the generalized phase matching condition, and examples of numerical data that is concerned with the performance of the chirped linear and curved grating lens will be presented in this paper.

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